

Polynomien sievennys

Tehtävät:

1. Sievennä.

a) $(a^2 - 2a) + (2a^2 + 5a - 2)$ b) $3x - 5 - (2 - 5x)$
c) $-(-x^2 - x + 3) + (x + 3)$ d) $(-y + 3) - (1 - 2y) - (-3y + 3)$

2. Vähennä trinomista $3x^2 - x + 6$ binomien $x^2 - 2x$ ja $-3x + 4$ erotus.

3. Sievennä.

a) $(x - 2)(x^2 + 2x + 2)$ b) $(5a^2 + 6a + 6)(a + 3)$
c) $2(a + b)(a + 2b) - (a - b)(a + 3b)$

4. Sievennä.

a) $x(x + 3) - (x - 3)(x + 2)$ b) $x^2(x - 2) - (x - 2)(x^2 - 2)$
c) $2x^4 \left(x - \frac{1}{3}\right) - \frac{1}{2}x(3 - x^2)$

5. Laske

a) $(5 + a)^2$ b) $(-b + 1)^2$ c) $(-c - 2)^2$
d) $\left(d + \frac{1}{2}\right)^2$ e) $\left(3e + \frac{1}{3}\right)^2$ f) $(f^2 + 2f)^2$

6. Ratkaise yhtälö.

a) $(x + 3)^2 + x(x - 2) - 2(x - 3)^2 = 7$
b) $19 - (y + 2)^2 = -2(y + 1)(y - 1) + (1 + y)^2$

7. Jaa tekijöihin

a) $y - xy^2$ b) $-2a - 2a^3$ c) $2x^4 + 8x^3 - 8x^2$

8. Jaa tekijöihin

a) $3a^2 + 6a$ b) $b^3 + 2b^2 + b$ c) $27c^3 - 12c$
d) $d^5 - d$ e) $e^5 - 10e^3 + 25e$ f) $f^2 + 2f + 2$

9. Sievännä

$$\text{a) } \frac{6a^2}{2a^5} \quad \text{b) } \frac{3x+3}{3x} \quad \text{c) } \frac{ab+b^2}{a^2+ab} \quad \text{d) } \frac{9a^2-1}{3a-1}$$

10. Sievännä

$$\text{a) } \frac{9a^2-6a+1}{9a-3} \quad \text{b) } \frac{2x+1}{x^2+x} - \frac{1}{x+1} \quad \text{c) } \frac{3}{x+1} - \frac{2x-1}{x^2+2x+1}$$

Ratkaisut:

1.

a) $(a^2 - 2a) + (2a^2 + 5a - 2) = a^2 - 2a + 2a^2 + 5a - 2 = 3a^2 + 3a - 2.$

b) $3x - 5 - (2 - 5x) = 3x - 5 - 2 + 5x = 8x - 7.$

c) $-(-x^2 - x + 3) + (x + 3) = x^2 + x - 3 + x + 3 = x^2 + 2x.$

d) $(-y + 3) - (1 - 2y) - (-3y + 3) = -y + 3 - 1 + 2y + 3y - 3 = 4y - 1.$

2.

$$\begin{aligned} & (3x^2 - x + 6) - ((x^2 - 2x) - (-3x + 4)) \\ &= (3x^2 - x + 6) - (x^2 - 2x + 3x - 4) \\ &= (3x^2 - x + 6) - (x^2 + x - 4) = 3x^2 - x + 6 - x^2 - x + 4 \\ &= 2x^2 - 2x + 10. \end{aligned}$$

3.

a) $(x - 2)(x^2 + 2x + 2) = x^3 + 2x^2 + 2x - 2x^2 - 4x - 4 = x^3 - 2x - 4$

b) $(5a^2 + 6a + 6)(a + 3) = 5a^3 + 15a^2 + 6a^2 + 18a + 6a + 18$
 $= 5a^3 + 21a^2 + 24a + 18$

c) $2(a + b)(a + 2b) - (a - b)(a + 3b)$
 $= 2(a^2 + 2ab + ba + 2b^2) - (a^2 + 3ab - ba - 3b^2)$
 $= 2a^2 + 4ab + 2ba + 4b^2 - a^2 - 3ab + ba + 3b^2 = a^2 + 4ab + 7b^2$

4.

a) $x(x + 3) - (x - 3)(x + 2) = (x^2 + 3x) - (x^2 + 2x - 3x - 6)$
 $= x^2 + 3x - x^2 - 2x + 3x + 6 = 4x + 6$

b) $x^2(x - 2) - (x - 2)(x^2 - 2) = (x^3 - 2x^2) - (x^3 - 2x - 2x^2 + 4)$
 $= x^3 - 2x^2 - x^3 + 2x + 2x^2 - 4 = 2x - 4$

c) $2x^4 \left(x - \frac{1}{3}\right) - \frac{1}{2}x(3 - x^2) = \left(2x^5 - \frac{2}{3}x^4\right) - \left(\frac{3}{2}x - \frac{1}{2}x^3\right)$
 $= 2x^5 - \frac{2}{3}x^4 + \frac{1}{2}x^3 - \frac{3}{2}x$

5.

- a) $(5 + a)^2 = 5^2 + 2 \cdot 5a + (2a)^2 = 25 + 10a + a^2$
- b) $(-b + 1)^2 = (-b)^2 + 2(-b) \cdot 1 + 1^2 = b^2 - 2b + 1$
- c) $(-c - 2)^2 = (-c)^2 + 2(-c) \cdot (-2) + (-2)^2 = c^2 + 4c + 4$
- d) $\left(d + \frac{1}{2}\right)^2 = d^2 + 2d \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 = d^2 + d + \frac{1}{4}$
- e) $\left(3e + \frac{1}{3}\right)^2 = (3e)^2 + 2 \cdot 3e \cdot \frac{1}{3} + \left(\frac{1}{3}\right)^2 = 9e^2 + 2e + \frac{1}{9}$
- f) $(f^2 + 2f)^2 = (f^2)^2 + 2f^2 \cdot 2f + (2f)^2 = f^4 + 4f^3 + 4f^2$

6. (a) Yhtälön vasen puoli on

$$\begin{aligned}(x + 3)^2 + x(x - 2) - 2(x - 3)^2 &= x^2 + 6x + 9 + x^2 - 2x - 2(x^2 - 6x + 9) \\ &= x^2 + 6x + 9 + x^2 - 2x - 2x^2 + 12x - 18 \\ &= 16x - 9\end{aligned}$$

Yhtälön ratkaisu on siis

$$16x - 9 = 7 \iff 16x = 16 \iff x = 1.$$

(a) Yhtälön vasen puoli on

$$19 - (y + 2)^2 = 19 - (y^2 + 4y + 4) = -y^2 - 4y + 15.$$

Yhtälön oikea puoli on

$$-2(y^2 - y + y - 1) + (1 + 2y + y^2) = -2y^2 + 2 + 1 + 2y + y^2 = -y^2 + 2y + 3.$$

Yhtälön ratkaisu on siis

$$-y^2 - 4y + 15 = -y^2 + 2y + 3 \iff -6y = -12 \iff y = 2.$$

7.

- a) $y - xy^2 = y(1 - xy)$
- b) $-2a - 2a^3 = -2a(1 + a^2)$
- c) $2x^4 + 8x^3 - 8x^2 = 2x^2(x^2 + 4x - 4)$

8.

- a) $3a^2 + 6a = 3a(a + 2)$
- b) $b^3 + 2b^2 + b = b(b^2 + 2b + 1) = b(b + 1)^2$
- c) $27c^3 - 12c = 3c(9c^2 - 4) = 3c(3c + 2)(3c - 2)$
- d) $d^5 - d = d(d^4 - 1) = d(d^2 + 1)(d^2 - 1) = d(d^2 + 1)(d + 1)(d - 1)$
- e) $e^5 - 10e^3 + 25e = e(e^4 - 10e^2 + 25) = e(e^2 - 5)^2$
- f) $f^2 + 2f + 2$ (ei jakaudu)

9.

- a) $\frac{6a^2}{2a^5} = \frac{2a^2 \cdot 3}{2a^2 \cdot a^3} = \frac{3}{a^3}$
- b) $\frac{3x + 3}{3x} = \frac{3(x + 1)}{3x} = \frac{x + 1}{x}$
- c) $\frac{ab + b^2}{a^2 + ab} = \frac{(a + b)b}{(a + b)a} = \frac{b}{a}$
- d) $\frac{9a^2 - 1}{3a - 1} = \frac{(3a + 1)(3a - 1)}{3a - 1} = 3a + 1$

10.

- a) $\frac{9a^2 - 6a + 1}{9a - 3} = \frac{(3a - 1)^2}{3(3a - 1)} = \frac{3a - 1}{3}$
- b) $\frac{2x + 1}{x^2 + x} - \frac{1}{x + 1} = \frac{2x + 1}{x(x + 1)} - \frac{x}{x(x + 1)} = \frac{2x + 1 - x}{x(x + 1)} = \frac{x + 1}{x(x + 1)} = \frac{1}{x}$
- c) $\frac{3}{x + 1} - \frac{2x - 1}{x^2 + 2x + 1} = \frac{3(x + 1)}{(x + 1)^2} - \frac{2x - 1}{(x + 1)^2} = \frac{3x + 3 - 2x + 1}{(x + 1)^2}$
 $= \frac{x + 4}{(x + 1)^2}$