Eksponentti- ja logaritmifunktioiden derivointi

Tehtävät:

1. Derivoi.

a) $3e^x$ b) $\frac{e^x}{3}$ c) e^{3x}

2. Derivoi.

a) e^{x+2} b) e^{-x} c) e^{1-x}

3. Derivoi.

a) e^{x^2} b) $e^{-\frac{x^2}{2}}$ c) $e^{\sqrt{x}}$

4. Derivoi.

a) $e^{2x} + e^{-2x}$ b) $\frac{1}{2}(e^x - e^{-x})$ c) $\frac{1 - e^x}{1 + e^x}$

5. Derivoi.

a) xe^x b) xe^{-x} c) $(e^x - 1)^5$

6. Derivoi.

a) $3 \ln x$ b) $\frac{\ln x}{3}$ c) $\ln 3x$

7. Derivoi.

a) $\ln \frac{x}{3}$ b) $\ln x^3$ c) $(\ln x)^3$

8. Derivoi.

a) $\ln(x + 2)$ b) $\ln(1 - x)$ c) $\ln |1 - x|$

9. Derivoi.

a) $\ln x^2$ b) $\ln \sqrt{x}$ c) $\ln \sqrt[3]{x}$

10. Derivoi.

a) $x \ln x$ b) $\frac{\ln x}{x}$ c) $\ln \ln x$

Ratkaisut:

1.

a) $D(3e^x) = 3 \cdot D\,e^x = 3e^x$

b) $D\left(\frac{e^x}{3}\right) = D\left(\frac{1}{3}e^x\right) = \frac{1}{3} \cdot D\,e^x = \frac{1}{3}e^x = \frac{e^x}{3}$

c) $D\,e^{3x} = e^{3x} \cdot D(3x) = e^{3x} \cdot 3 = 3e^{3x}$

2.

a) $D\,e^{x+2} = e^{x+2} \cdot D(x+2) = e^{x+2} \cdot 1 = e^{x+2}$

b) $D\,e^{-x} = e^{-x} \cdot D(-x) = e^{-x} \cdot (-1) = -e^{-x}$

c) $D\,e^{1-x} = e^{1-x} \cdot D(1-x) = e^{1-x} \cdot (-1) = -e^{1-x}$

3.

a) $D\,e^{x^2} = e^{x^2} \cdot D(x^2) = e^{x^2} \cdot 2x = 2xe^{x^2}$

b) $D\,e^{-\frac{x^2}{2}} = e^{-\frac{x^2}{2}} \cdot D\left(-\frac{x^2}{2}\right) = e^{-\frac{x^2}{2}} \cdot \left(-\frac{2x}{2}\right) = -xe^{-\frac{x^2}{2}}$

c) $D\,e^{\sqrt{x}} = e^{\sqrt{x}} \cdot D(\sqrt{x}) = e^{\sqrt{x}} \cdot \left(\frac{1}{2}x^{-1/2}\right) = e^{\sqrt{x}} \cdot \left(\frac{1}{2\sqrt{x}}\right) = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

4.

a) $D(e^{2x} + e^{-2x}) = e^{x^2} \cdot 2 + e^{-2x} \cdot (-2) = 2e^{x^2} - 2e^{-2x}$

b) $D\left(\frac{1}{2}(e^x - e^{-x})\right) = \frac{1}{2}(e^x - e^{-x} \cdot (-1)) = \frac{1}{2}(e^x + e^{-x})$

c) $D\left(\frac{1-e^x}{1+e^x}\right) = \frac{D(1-e^x) \cdot (1+e^x) - (1-e^x) \cdot D(1+e^x)}{(1+e^x)^2}$

$= \frac{-e^x \cdot (1+e^x) - (1-e^x) \cdot e^x}{(1+e^x)^2} = \frac{-e^x - (e^x)^2 - (e^x - (e^x)^2)}{(1+e^x)^2}$

$= \frac{-e^x - (e^x)^2 - e^x + (e^x)^2}{(1+e^x)^2} = \frac{-2e^x}{(1+e^x)^2}$

5.

a) $D(xe^x) = D\,x \cdot e^x + x \cdot D\,e^x = 1 \cdot e^x + x \cdot e^x = e^x + xe^x = (1+x)e^x$

b) $D(xe^{-x}) = D\,x \cdot e^{-x} + x \cdot D\,e^{-x} = 1 \cdot e^{-x} + x \cdot (-e^{-x}) = e^{-x} - xe^{-x}$
$= (1-x)e^{-x}$

c) $D(e^x - 1)^5 = 5(e^x-1)^4 \cdot D(e^x - 1) = 5(e^x-1)^4 \cdot e^x = 5e^x(e^x-1)^4$

6.

$$\text{a)}\quad D(3\ln x) = 3\cdot D\ln x = 3\cdot\frac{1}{x} = \frac{3}{x}$$

$$\text{b)}\quad D\left(\frac{\ln x}{3}\right) = D\left(\frac{1}{3}\ln x\right) = \frac{1}{3}\cdot D(\ln x) = \frac{1}{3}\cdot\frac{1}{x} = \frac{1}{3x}$$

$$\text{c)}\quad D(\ln 3x) = \frac{1}{3x}\cdot D(3x) = \frac{1}{3x}\cdot 3 = \frac{3}{3x} = \frac{1}{x}$$

Huom. c)-kohdan perusteella $D(\ln x) = D(\ln 3x)$. Integraalilaskennan peruslauseen mukaan täytyy päteä $\ln 3x = \ln x + C$, missä C on joku vakio. Näin onkin, sillä $\ln 3x = \ln 3 + \ln x$.

7.

$$\text{a)}\quad D\left(\ln\frac{x}{3}\right) = D\left(\ln\left(\frac{1}{3}\cdot x\right)\right) = D\left(\ln\frac{1}{3} + \ln x\right) = 0 + \frac{1}{x} = \frac{1}{x}$$

$$\text{b)}\quad D\ \ln x^3 = D(3\ln x) = 3\cdot\frac{1}{x} = \frac{3}{x}$$

$$\text{c)}\quad D(\ln x)^3 = 3(\ln x)^2\cdot D\ \ln x = 3(\ln x)^2\cdot\frac{1}{x} = \frac{3(\ln x)^2}{x}$$

8.

$$\text{a)}\quad D\ \ln(x+2) = \frac{1}{x+2}\cdot D(x+2) = \frac{1}{x+2}\cdot 1 = \frac{1}{x+2}$$

$$\text{b)}\quad D\ \ln(1-x) = \frac{1}{1-x}\cdot D(1-x) = \frac{1}{1-x}\cdot(-1) = \frac{1}{x-1}$$

c) Funktio $f(x) = \ln|1-x|$ on määritelty, kun $x < 1$ tai kun $x > 1$. Edellisessä tapauksessa $f(x) = \ln(1-x)$, jolloin b)-kohdan mukaan

$$f'(x) = \frac{1}{x-1}.$$

Toisaalta, kun $x > 1$, niin $f(x) = \ln(x-1)$, jolloin

$$f'(x) = \frac{1}{x-1}\cdot D(x-1) = \frac{1}{x-1}\cdot 1 = \frac{1}{x-1}.$$

Joka tapauksessa siis $f'(x) = \dfrac{1}{x-1}$.

9.

$$\text{a)}\quad D\ \ln x^2 = D(2\ln x) = 2\frac{1}{x} = \frac{2}{x}$$

$$\text{b)}\quad D\ \ln\sqrt{x} = D\ \ln x^{1/2} = D\left(\frac{1}{2}\ln x\right) = \frac{1}{2}\cdot\frac{1}{x} = \frac{1}{2x}$$

$$\text{c)}\quad D\ \ln\sqrt[3]{x} = D\ \ln x^{1/3} = D\left(\frac{1}{3}\ln x\right) = \frac{1}{3}\cdot\frac{1}{x} = \frac{1}{3x}$$

10.

$$\text{a)}\quad D(x\ln x) = D\,x\cdot\ln x + x\cdot D\ \ln x = 1\cdot\ln x + x\cdot\frac{1}{x} = \ln x + 1$$

$$\text{b)}\quad D\left(\frac{\ln x}{x}\right) = \frac{D\ \ln x\cdot x - \ln x\cdot D\,x}{x^2} = \frac{\frac{1}{x}\cdot x - \ln x\cdot 1}{x^2} = \frac{1-\ln x}{x^2}$$

$$\text{c)}\quad D\ \ln\ln x = \frac{1}{\ln x}\cdot D\ \ln x = \frac{1}{\ln x}\cdot\frac{1}{x} = \frac{1}{\ln x}\cdot\frac{1}{x} = \frac{1}{x\ln x}$$