Y100 (Mathematics I)
Exercise 6 (last exercise, 2 pages)
10.–13.12.2007

1. Solve the following systems of equations by elimination in matrix form:

$$\text{a)} \left\{ \begin{array}{rrrcr} 2x & +y & 2x & = & -2 \\ & y & -2z & = & 4 \\ x & -y & 3z & = & -2 \end{array} \right. , \qquad \text{b)} \left\{ \begin{array}{rrrcr} x_1 & +x_2 & +4x_3 & = & -2 \\ -x_1 & +x_2 & -2x_3 & = & 4 \\ 2x_1 & -x_2 & +5x_3 & = & 0 \end{array} \right. .$$

2. On both sides of a chemical reaction equation there must appear an equal number of atoms of each element. In the following equations the coefficients of each compound are unknown (x_1, x_2, etc.). Write out the systems of equations that these coefficients must satisfy. Then solve these systems by elimination. If you get an infinite amount of solutions, choose the one you think is the most suitable for the reaction equation. What happens if the solution is unique?
   a) $x_1\mathrm{CO_2} + x_2\mathrm{H_2O} \quad \rightarrow \quad x_3\mathrm{O_2} + x_4\mathrm{C_6H_{12}O_6}$,
   b) $x_1\mathrm{CaCO_3} + x_2\mathrm{HCl} \quad \rightarrow \quad x_3\mathrm{CaCl_2} + x_4\mathrm{CO_2} + x_5\mathrm{H_2O}$,
   c) $x_1\mathrm{MgS}+x_2\mathrm{H_2O} \quad \rightarrow \quad x_3\mathrm{SO_2} + x_4\mathrm{Mg}$.

   Hint. For example, in part a) there are x_1 carbon atoms on the left side and $6x_4$ carbon atoms on the right side. They must therefore satisfy the equation $x_1 = 6x_4$, which can also be written as $x_1 - 6x_4 = 0$. In the same way we get the equation for oxygen atoms: $2x_1 + x_2 - 2x_3 - 6x_4 = 0$, and the one for hydrogen atoms: $2x_2 - 12x_4 = 0$.

3. Let

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 5 & 0 \\ -1 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -2 & 0 \\ -4 & 3 & 2 \\ 3 & -1 & 0 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix}.$$

   Calculate those matrices that are definable: $A+B$, $B-C$, BC, CB, CA, C^2, $C(A+B)$.

4. Show by multiplication that matrices A and B are each others' inverses, when

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 3 & -1 & 1 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B = \frac{1}{4} \begin{bmatrix} 1 & 1 & -2 & -5 \\ 3 & -1 & -6 & -11 \\ 3 & -1 & -10 & -15 \\ -3 & 1 & 10 & 19 \end{bmatrix}.$$

   Compute also the matrix ABA.

5. In the following, find the matrix inverse of the given matrix or show that the matrix is not invertible:

$$\text{(a)} \quad A = \begin{bmatrix} 1 & 3 & 3 \\ -1 & -2 & 0 \\ 2 & 6 & 5 \end{bmatrix}, \qquad \text{(b)} \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

6. The incoming solar power hitting a power detector was observed three times a day: at 1.00 pm, at 2.00 pm and at 3.00 pm. The observation was repeated on three days, and the results were collected into the following table.

	1.00	2.00	3.00
day 1	31 W	30 W	27 W
day 2	30 W	30 W	28 W
day 3	30 W	29 W	28 W

A parabola will be now fitted to the values obtained each day (cf. Example 5.9 in the lectures, altogether we shall need three different parabolas). The time values 1, 2 and 3 are used as values for the x-coordinate. On the first day, for example, the parabola needs to pass through the points $(1,31)$, $(2,30)$ and $(3,27)$.

The equation of a parabola is $y = ax^2 + bx + c$. For each day, form the corresponding system of equations to determine the coefficients a, b and c, and write each system as a matrix equation $AX = B$. The coefficient matrix A will be the same for each day. Find the matrix inverse for the coefficient matrix, and solve the equations for each parabola with the help of that inverse.

7. Give course feedback via a questionnaire that will appear on the course's web page. (You may also complete this assignment after the exam, as long as you won't forget it.)