Y100 (Mathematics 1)
Exercise 4 (2 pages)
26.–29.11.2007

1. Calculate $\ln 10$, $\lg 7$, $\log_6 9$ and $\ln \sqrt{10^{1000}}$ with the help of a calculator. Estimate also how many digits there are in the number $4{,}5^{100}$. (Use the rules governing exponents and logarithms. Ten-based logarithm gives approximately the amount of digits in a number.)

2. Find the greatest value of the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = xe^{-x}$.

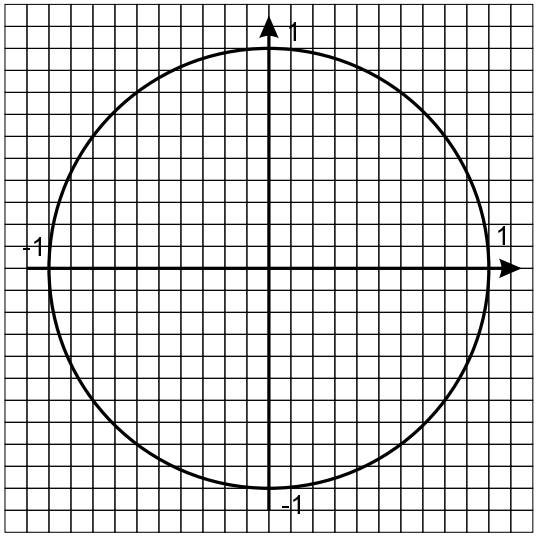
   Hint. You'll need the differentiation rules for products and composed functions. Imitate the example in the lectures. The derivative should be $e^{-x}(1 - x)$. Remember also that e^{-x} is always positive.

3. Compute the following integrals:

   (a) $\displaystyle\int_1^e \frac{2}{x} - \frac{1}{x^2}\,dx$, (b) $\displaystyle\int_0^2 \frac{1}{2x+1}\,dx$, (c) $\displaystyle\int_0^{\ln 2} e^{3x}\,dx$.

   Hint. In parts (a) and (b), write all terms in a form using powers only (that is, $1/x^2 = x^{-2}$ and so forth). In parts (b) and (c) you'll need the integration rule concerning the derivative of a composed function (rule number 5, see also example 5.8). For example, in part (c) multiply the integrand by three and the whole integral by $1/3$ to get inside the integral the derivative of the inner function, which you need.

4. Draw inside the unit circle the following directed angles: $45°$, $200°$ and $-30°$. Convert these angles to radians. Solve then by approximating from the same picture the values $\sin 45°$, $\cos 45°$, $\sin -30°$ and $\cos 200°$. Check the results with a calculator.



5. Differentiate the following functions:

   (a) $f(x) = e^x \sin x$, (b) $g(x) = e^{(x^2)}$, (c) $h(x) = \dfrac{\cos x}{\sin x}$.

6. The photosynthesis rate of phytoplankton was examined by observing the fixation of carbon-14 isotope in a northern lake. The efficiency of the photosynthesis varied periodically along seasons, and the fixation of carbon-14 in the whole lake was observed to follow approximately the function

   $$s(t) = 300 \cos\left(\frac{\pi}{6} \cdot t\right) + 500 \quad \text{kg/kk},$$

   where t denotes time in months from the beginning of July. Find out when the photosynthesis rate reaches its minimum, and compute by integrating how much carbon-14 was fixed during the summer (in June, July and August).