Algebra II
Department of Mathematics and Statistics
Problem sheet 3
Thu 11.2.2010

1. Describe the conjugacy classes of the groups S_4 and A_4, and find their sizes. Find all normal subgroups of these groups.

2. Assume that in the representation of $\sigma \in S_n$ as a product of disjoint cycles, there are cycles of the same length or cycles of even length. Show that there exists $\tau \notin A_n$, such that ${}^{\tau}\sigma = \sigma$ (that is, τ centralises σ).

3. Find the conjugacy classes and the centre of the dihedral group D_{2n}.

   Hint: Every element of D_{2n} can be written as ρ^j or $\rho^j\sigma$. The conjugacy classes should represent different rotation angles and different types of axes of reflection.

4. Assume that G acts on the set X, and that $g, h \in G$ belong to the same conjugacy class. Prove that $|\operatorname{Fix}(g)| = |\operatorname{Fix}(h)|$.

5. Count the number of distinct colourings of the edges of a regular hexagon, using 4 colours.

6. Prove that the symmetry group of a cube is isomorphic to S_4, and verify that the conjugacy classes of the said symmetry group are determined by axes and angles of rotation, as was mentioned in Example 3.12.

   Hint: Define the natural action of the symmetry group of a cube on the set of diagonals passing through opposite corners. Show that this action in a one-to-one homomorphism to the group S_4 (so called *faithful* action). To prove surjectivity, you can count the size of the symmetry group with the help of Theorem 2.7 (considering a point stabiliser).