

**Motivation:** The study of **computational models of argumentation** is an active and vibrant area of modern **AI** research. **Incomplete argumentation frameworks (IAFs)** generalize Dung's argumentation frameworks for **reasoning under uncertainty**.

## INCOMPLETE ARGUMENTATION FRAMEWORKS

### Argumentation Framework (AF)

A directed graph  $AF = (A, R)$ :

- $A$  is the set of **arguments**
- $R \subseteq A \times A$  is the **attack relation**

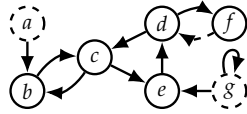
### Semantics $\sigma$ define $\sigma$ -extensions

- jointly acceptable sets of arguments
- required to be **conflict-free**
- other desired properties give rise to different semantics: admissible, stable, complete, grounded, preferred

### Incomplete Argumentation Framework (IAF)

$IAF = (A, A^?, R, R^?)$ , where

- $A$  and  $R$  are **definite** arguments and attacks,
- $A^?$  and  $R^?$  are **uncertain** arguments and attacks,
- $R, R^? \subseteq (A \cup A^?) \times (A \cup A^?)$ .



### Acceptance Problems in IAFs

**Completion:** standard AF containing the fixed part and a valid subset of uncertain elements

An argument  $a \in A$  is

- **credulously** accepted (CA):  $a$  is in **some** extension
- **skeptically** accepted (SA):  $a$  is in **all** extensions
- + existence of an extension (ExSA)

Acceptance of  $a \in A$  holds

- **possibly** (P) if it holds for **some** completion
- **necessarily** (N) if it holds for **all** completions

## SAT-BASED ALGORITHMS

**Input:**  $IAF = (A, A^?, R, R^?), a \in A$ .

**Basis:** **SAT Encodings**

### Boolean variables:

- $y_a$  for each argument  $a$ ,  $r_{a,b}$  for each attack  $(a, b)$ 
  - true iff element included in completion
- $x_a$  for each argument  $a$ 
  - true iff argument included in  $\sigma$ -extension
- $z_a$  for each argument  $a$ 
  - true iff argument attacked by  $\sigma$ -extension

### Propositional formulas:

- $\varphi_{\sigma}(CAF)$  encodes valid completions
- $\varphi_{\sigma}(CAF)$  for  $\sigma \in \{AD, ST, CP, GR\}$  encodes the semantics

The encoding for complete semantics is

$$\varphi_{CP}(IAF) = \varphi_{AD}(IAF) \wedge \bigwedge_{a \in A \cup A^?} \left( \bigwedge_{(b,a) \in R \cup R^?} \left( (y_a \wedge y_b \wedge r_{b,a}) \rightarrow z_b \right) \rightarrow x_a \right).$$

PCA and NSA solved via direct SAT calls.

### SAT-based CEGAR Algorithms

Algorithms where a **SAT solver** is called **iteratively** and incrementally using different assumptions.

**Task:** Possible skeptical acceptance under preferred.

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 $\varphi \leftarrow \text{ABSTRACTION}(IAF, a) \quad \triangleright$  Initialize abstraction
while true do
   $(sat, \tau) \leftarrow \text{SAT}(\varphi) \quad \triangleright$  Solve abstraction
  if  $sat = false$  then return reject
   $AF^* \leftarrow \text{COMPLETION}(\tau)$ 
   $(sat, \tau) \leftarrow \text{CEGAR}(\text{CHECK}(IAF, AF^*, a)) \quad \triangleright$  Counter-
  if  $sat = false$  then return accept  $\triangleright$  example?
   $\varphi \leftarrow \varphi \wedge \text{REFINE}(IAF, AF^*) \quad \triangleright$  Exclude completion
end while

```

**Strong refinement:** take into account which atomic changes preserve the counterexample extension of the completion: e.g. for preferred semantics adding arguments which are attacked by the extension can be safely ignored.

**In journal article:** SAT encodings, CEGAR algorithms, and strong refinements for all other variants

**Contributions:** **Complexity analysis** of acceptance problems in IAFs under various central AF semantics. **Design of algorithms** for solving concrete instances of acceptance problems: direct **SAT encodings** and SAT-based **CEGAR algorithms**. **Strong refinements** for CEGAR algorithms by analyzing redundant atomic changes in IAFs. **System implementation** and **empirical evaluation:** SAT-based approach viable in practice.

## COMPUTATIONAL COMPLEXITY

s	s-PCA	s-NCA	s-PSA (s-PEXSA)	s-NSA (s-NEXSA)
CF	$\in P$	$\in P$	trivial	trivial
$CF_{\neq \emptyset}$	$\in P$	$\in P$	$\in P$	$\in P$
AD	NP-c.	$\Pi_2^p$ -c.	trivial	trivial
$AD_{\neq \emptyset}$	NP-c.	$\Pi_2^p$ -c.	$\Sigma_2^p$ -c.	coNP-c. ( $\Pi_2^p$ -c.)
ST	NP-c.	$\Pi_2^p$ -c.	$\Sigma_2^p$ -c.	coNP-c. ( $\Pi_2^p$ -c.)
CP	NP-c.	$\Pi_2^p$ -c.	NP-c.	coNP-c.
GR	NP-c.	coNP-c.	NP-c.	coNP-c.
PR	NP-c.	$\Pi_2^p$ -c.	$\Sigma_3^p$ -c.	$\Pi_2^p$ -c.

- Lower bounds follow from quantifier representations of the problem
    - Possible:  $\exists$  completion
    - Necessary:  $\forall$  completions
  - Upper bounds via reductions from quantified satisfiability
- In journal article:** full proofs and illustrations of reductions

## EMPIRICAL EVALUATION

**Implementation:** **TAEDENNAE**

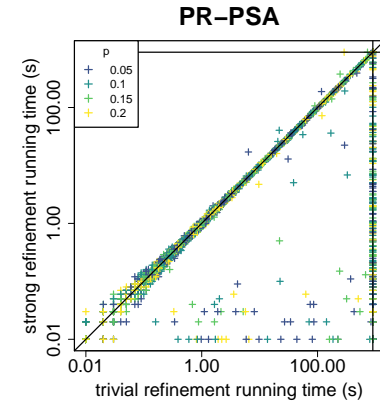
- Solver for acceptance in incomplete AFs
- Direct SAT calls and SAT-based CEGAR algorithms
- Makes use of incremental SAT solving
- Available online in open source:

[bitbucket.org/andreasniskanen/taeydennae](http://bitbucket.org/andreasniskanen/taeydennae)

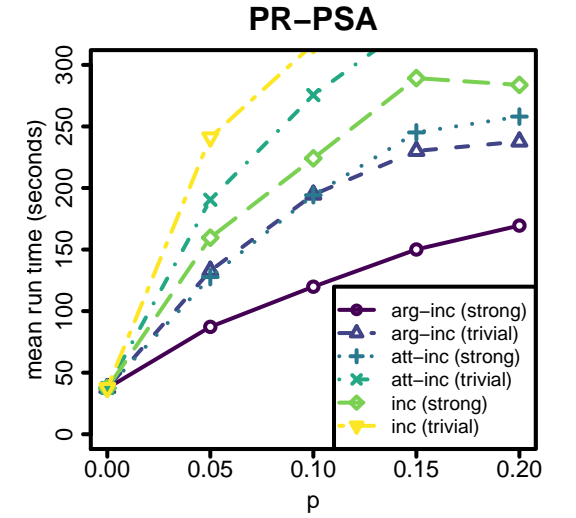
**Benchmark instances**

4200 IAFs generated from ICCMA'17 AFs

- Parameter: probability  $p$  of element being uncertain



**Task:** Possible skeptical acceptance under preferred



**In journal article:** Empirical results for other variants, comparison to basic enumeration-based approach