TECHNISCHE UNIVERSITÄT WIEN
UNIVERSITÀ DEGLI STUDI DI GENOVA
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# Novel Algorithms for Abstract Dialectical Frameworks based on Complexity Analysis of Subclasses and SAT Solving

Thomas Linsbichler  Marco Maratea  Andreas Niskanen  Johannes P. Wallner  Stefan Woltran

**Motivation:** The study of **computational aspects of argumentation** is an active area of modern AI research.
**Abstract dialectical frameworks** are a powerful generalization of Dung's **argumentation frameworks**.
Expressive power comes with a price: **computational complexity** one level higher on the polynomial hierarchy.

**Contributions:**
-**Complexity analysis** of ADF subclasses: $k$-bipolar, ($k$-)acyclic, and ($k$-)concise
-**Design of algorithms** for acceptance problems based on incremental SAT solving
-**Implementation** and empirical evaluation

## ABSTRACT DIALECTICAL FRAMEWORKS: DEFINITIONS

### Syntax of ADFs

A tuple $D = (A, L, C)$, where

- $A$ is a finite set of **arguments**,
- $L \subseteq A \times A$ is a set of **links**,
- $C = \{\varphi_a\}_{a \in A}$ is a set of **acceptance conditions**: each $\varphi_a$ is a propositional formula over the parents of $a$.

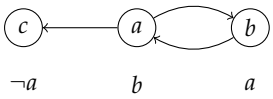



Figure 1: Example ADF.

### Semantics of ADFs

An interpretation $I$ maps each argument to a truth value in $\{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$. Let $I \leq_i J$ if $I(a) \in \{\mathbf{t}, \mathbf{f}\}$ implies $I(a) = J(a)$ for all $a \in A$.
$I$ is *admissible*, $I \in adm(D)$, if for all $a \in A$

- $I(a) = \mathbf{t}$ implies $\varphi_a[I]$ is a tautology,
- $I(a) = \mathbf{f}$ implies $\varphi_a[I]$ is unsatisfiable,

where $\varphi_a[I]$ is the formula obtained from $\varphi_a$ by replacing each argument that $I$ assigns to $\mathbf{t}$ or $\mathbf{f}$ with $\top$ and $\bot$.
$I$ is *preferred*, $I \in prf(D)$, if it is $\leq_i$-maximal admissible.

### ADF Reasoning Tasks

Let $\sigma$ be an ADF semantics.

| | Input | Decision |
|---|---|---|
| $Cred_\sigma$ | $D, a \in A$ | $\exists I \in \sigma(D), I(a) = \mathbf{t}$? |
| $Skept_\sigma$ | $D, a \in A$ | $\forall I \in \sigma(D), I(a) = \mathbf{t}$? |
| $Exists_\sigma^>$ | $D, I$ | $\exists J \in \sigma(D), J >_i I$? |
| $Ver_\sigma$ | $D, I$ | $I \in \sigma(D)$? |

In Figure 1, argument $a$ is not skeptically accepted under preferred, since $I$ with $I(a) = \mathbf{f}$, $I(b) = \mathbf{f}$, $I(c) = \mathbf{t}$ is preferred.

## COMPUTATIONAL COMPLEXITY OF SUBCLASSES

An ADF is bipolar if every link is *attacking* or *supporting*.

An ADF is $k$-bipolar if for every $a \in A$, there are at most $k$ links $(b, a) \in L$ that are neither attacking nor supporting.

| | ADFs | | | | $k$-bipolar ADFs | | | |
|---|---|---|---|---|---|---|---|---|
| $\sigma$ | $Cred_\sigma$ | $Skept_\sigma$ | $Exists_\sigma^>$ | $Ver_\sigma$ | $Cred_\sigma$ | $Skept_\sigma$ | $Exists_\sigma^>$ | $Ver_\sigma$ |
| *cf* | NP-c | trivial | NP-c | NP-c | in P | trivial | in P | in P |
| *nai* | NP-c | $\Pi_2^P$-c | NP-c | DP-c | in P | coNP-c | in P | in P |
| *adm* | $\Sigma_2^P$-c | trivial | $\Sigma_2^P$-c | coNP-c | NP-c | trivial | NP-c | in P |
| *grd* | coNP-c | coNP-c | coNP-c | DP-c | in P | in P | in P | in P |
| *com* | $\Sigma_2^P$-c | coNP-c | $\Sigma_2^P$-c | DP-c | NP-c | in P | NP-c | in P |
| *prf* | $\Sigma_2^P$-c | $\Pi_3^P$-c | $\Sigma_2^P$-c | $\Pi_2^P$-c | NP-c | $\Pi_2^P$-c | NP-c | coNP-c |

Complexity of general [Strass and Wallner, 2015] and $k$-bipolar ADFs.

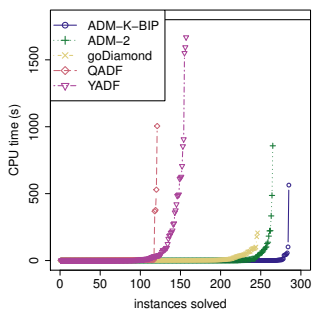
## SAT-BASED ALGORITHMS FOR ACCEPTANCE IN ADFS

- Complexity-sensitive algorithms for skeptical and credulous acceptance under preferred semantics
  - Detect whether input ADF is $k$-bipolar for small enough $k$
- Utilize SAT solvers as the main search engine
- System k++ADF implementing the algorithms available at `www.cs.helsinki.fi/group/coreo/k++adf`

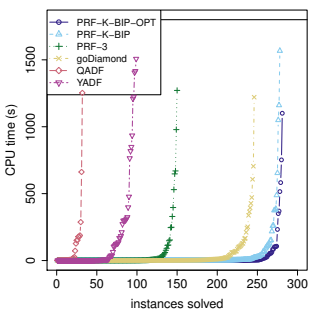
Skeptical acceptance under preferred for $k$-bipolar ADFs:

- Suitable NP fragment for a SAT solver is $Exists_{adm}^>$
- The resulting admissible interpretation $I$ can be extracted from the truth assignment
- Search for preferred interpretations by iteratively solving $Exists_{adm}^>(D, I)$ and setting $I$ as the corresponding interpretation
- If the query argument is not assigned to true, we can reject it — otherwise, rule out all interpretations $J \leq_i I$ from the search space and continue

## EMPIRICAL EVALUATION




Credulous admissible




Skeptical preferred

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