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# Deciding Acceptance in Incomplete Argumentation Frameworks

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**Motivation:** The study of **computational models of argumentation** is an active and vibrant area of modern **AI** research.
**Incomplete argumentation frameworks** generalize Dung's argumentation frameworks for **reasoning under uncertainty**.
**Algorithmic techniques for deciding acceptance** in incomplete argumentation frameworks have not been studied to date.

**Contributions:**
**Complexity analysis** of new variants of skeptical acceptance: exclude nonempty (sets of) extensions to avoid counterintuitive solutions
**Design of algorithms** for acceptance in IAFs based on SAT solving: make use of observations regarding redundant changes in IAFs
**Implementation** and empirical evaluation: promising results in terms of practical performance

## INCOMPLETE ARGUMENTATION FRAMEWORKS

### Argumentation Framework (AF)

A directed graph $AF = (A, R)$, where
- $A$ is the set of **arguments**
- $R \subseteq A \times A$ is the **attack relation**

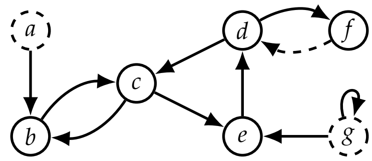
**Semantics** define **extensions**
- Required to be **conflict-free** (CF)
- $S \in \text{CF}(AF)$ is **admissible** (AD) if $S$ attacks every attacker of $S$
- $S \in \text{CF}(AF)$ is **stable** (ST) if $S$ attacks everything outside $S$

Argument accepted **credulously** (CA) if it is in **some** extension, **skeptically** (SA) if it is in **all** extensions

### Incomplete Argumentation Framework (IAF)

A tuple $IAF = (A, A^?, R, R^?)$, where
- $A$ and $R$ are **definite** arguments and attacks
- $A^?$ and $R^?$ are **uncertain** arguments and attacks

A standard AF containing all definite elements and any uncertain elements is called a **completion**




Example incomplete argumentation framework.

### Argument Acceptance in IAFs

Acceptance of an argument holds **possibly** (PCA,PSA) if it holds in **some** completion, **necessarily** (NCA, NSA) if it holds in **all** completions

**s-PExSA**

Does there exist a completion $AF^*$ of $IAF$ such that $AF^*$ has an **s** extension and for each **s**-extension $E$ of $AF^*$, $a \in E$?

SA is **trivial** under CF and AD: **excluding empty set** from the set of extensions results in $\text{CF}_{\neq\emptyset}$ and $\text{AD}_{\neq\emptyset}$

## COMPUTATIONAL COMPLEXITY

| | s-ExSA | s-PSA | s-PExSA | s-NSA | s-NExSA |
|---|---|---|---|---|---|
| $\text{CF}_{\neq\emptyset}$ | **in P** | **in P** | **in P** | **in P** | **in P** |
| $\text{AD}_{\neq\emptyset}$ | **DP-c.** | $\mathbf{\Sigma_2^P}$**-c.** | $\mathbf{\Sigma_2^P}$**-c.** | coNP-c. | $\mathbf{\Pi_2^P}$**-c.** |
| ST | DP-c. | $\Sigma_2^P$-c. | $\mathbf{\Sigma_2^P}$**-c.** | coNP-c. | $\mathbf{\Pi_2^P}$**-c.** |

- Reasoning under $\text{CF}_{\neq\emptyset}$ is always tractable
- No complexity jump from PSA to PExSA: problem remains complete for second level
- For NExSA second-level completeness: in contrast to first-level completeness for NSA

## SAT-BASED ALGORITHMS FOR ACCEPTANCE IN IAFS

Input: $IAF = (A, A^?, R, R^?)$, $a \in A$, $\mathbf{s} \in \{\text{AD}_{\neq\emptyset}, \text{ST}\}$
For **s**-PCA and **s**-NSA, a single call to a SAT solver suffices.

$$\varphi_?(IAF) = \bigwedge_{a\in A} y_a \wedge \bigwedge_{(a,b)\in R} r_{a,b} \wedge \bigwedge_{a\in A^?} \left(\neg y_a \rightarrow \left(\neg x_a \wedge \bigwedge_{(a,b)\in R^?} \neg r_{a,b} \wedge \bigwedge_{(b,a)\in R^?} \neg r_{b,a}\right)\right)$$

$$\varphi_{\text{AD}}(IAF) = \varphi_{\text{CF}}(IAF) \wedge \bigwedge_{a\in A\cup A^?} \bigwedge_{(b,a)\in R\cup R^?} \left((x_a \wedge y_a \wedge y_b \wedge r_{b,a}) \rightarrow \bigvee_{(c,b)\in R\cup R^?} (x_c \wedge y_c \wedge r_{c,b})\right)$$

$$\varphi_{\text{ST}}(IAF) = \varphi_{\text{CF}}(IAF) \wedge \bigwedge_{a\in A\cup A^?} \left((\neg x_a \wedge y_a) \rightarrow \bigvee_{(b,a)\in R\cup R^?} (x_b \wedge y_b \wedge r_{b,a})\right)$$

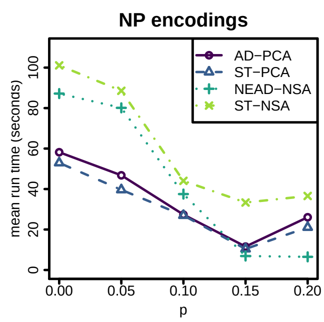
$\varphi_?(IAF) \wedge \varphi_{\mathbf{s}}(IAF) \wedge x_a$ is SAT iff **s**-PCA is *accept*
$\varphi_?(IAF) \wedge \varphi_{\mathbf{s}}(IAF) \wedge \neg x_a$ is UNSAT iff **s**-NSA is *reject*

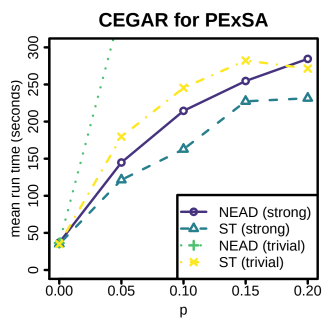
For **s**-PExSA: a SAT-based counterexample-guided abstraction refinement (CEGAR) procedure, where a SAT solver is called iteratively and incrementally

```
φ ← φ?(IAF) ∧ φs(IAF) ∧ xa                  ▷ initialize abstraction
while true do
    (sat, τ) ← SAT(φ)                       ▷ solution to abstraction?
    if sat = false then return reject       ▷ UNSAT
    AF* ← EXTRACT(τ)                        ▷ get candidate completion
    (sat, τ) ← SAT(φs(AF*) ∧ ¬xa)           ▷ counterexample?
    if sat = false then return accept       ▷ UNSAT
    φ ← φ∧ REFINE(IAF, AF*)                 ▷ exclude completion
end while
```

**Strong refinement:** instead of excluding the current completion $AF^*$, take into account also those atomic changes which still preserve the counterexample extension

## EMPIRICAL EVALUATION




## REFERENCES

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