Metabolic Modeling, Ex Tempore exercises, Introduction to Matlab

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In these exercises we familiarize ourselves to MATLAB environment and train Gaussian elimination You can find help for using MATLAB

- from Jarmo Hurri's slides for the course computational data analysis `http://www.cs.helsinki.fi/u/hurri/opetus/lda-I/tiedostot/matlab.pdf`;
- by typing '*help command*' in the MATLAB environment, where 'command' is the name of the function you want to know more about, or
- from the on-line help of MATLAB `http://www.mathworks.com/access/helpdesk/help/techdoc/`.

1. By using pen and paper, solve the following system of linear equations by transforming a corresponding augmented matrix to reduced row echelon form. Use Gauss-Jordan reduction.

$$\begin{array}{ccccl} 3x_1 & +x_2 & -2x_3 & +2x_4 & = 3 \\ 2x_1 & +4x_2 & +x_3 & -2x_4 & = -1 \\ x_1 & +x_2 & +x_3 & +x_4 & = 6 \\ & -x_2 & -x_3 & +x_4 & = 0 \end{array}$$

2. Verify the result of the previous exercise in MATLAB. Type the coefficient matrix A and vector b corresponding the above linear equation system $\mathbf{Ax} = \mathbf{b}$ to MATLAB. Check the rank of $\mathbf{A}$ (MATLAB command *rank*). Construct an augmented matrix $[A|b]$ by concatenating $\mathbf{b}$ to $\mathbf{A}$. Call function *rref* to transform the augmented matrix to reduced row echelon form. Extract vector $\mathbf{x}$ from the augmented matrix and assign it some variable (operator *:*, function *end*).

3. Construct random, 200×200 matrix $\mathbf{A}$ and random vector $\mathbf{b} \in \mathbf{R}^{200}$, both having integer entries from the interval [0,10] (hint: *rand, round*). Solve the system $\mathbf{Ax} = \mathbf{b}$ by

   (a) applying \ operator (*help mldivide*),

   (b) by constructing the augmented matrix and transforming it to reduced row echelon form, and

   (c) by computing the inverse (*inv*) $\mathbf{A}^{-1}$ of $\mathbf{A}$ and then $\mathbf{A}^{-1}\mathbf{b}$, Record cpu-time used by the operations (*cputime*). Compute the norms of the residual vectors $\mathbf{r} = \mathbf{A}\mathbf{x} - \mathbf{b}$ for each method (*norm*). Which method is the fastest? Which one gives the most accurate results?

4. By using pen and paper, solve the following system of linear equations by transforming it to reduced row echelon form. Use Gauss-Jordan reduction.

$$\begin{array}{lllll} 3x_1 & +x_2 & -x_3 & -x_4 & = 1 \\ 2x_1 & -x_2 & -2x_3 & -x_4 & = 0 \\ -x_1 & +x_2 & -x_3 & +3x_4 & = 0 \end{array}$$

   Verify the result using MATLAB. How would you interpret the result? Solve the system in MATLAB using \ operator and Moore-Penrose pseudo inverse *pinv*. Compare the results obtained with different methods. Check also the norms of vectors $\mathbf{x}$. How would you interpret the result? Compute the basis of the null space of coefficient matrix $\mathbf{A}$ (*null*). Select some vector $\mathbf{n}$ from the null space and verify that $\mathbf{A} * \mathbf{n} = 0$.

5. By using MATLAB, try to solve the following system of linear equations by transforming it to reduced row echelon form.

$$\begin{array}{lllll} 4x_1 & +2x_2 & -3x_3 & +3x_4 & = 4 \\ 2x_1 & +4x_2 & +x_3 & -2x_4 & = -1 \\ x_1 & +x_2 & +x_3 & +x_4 & = 6 \\ & -x_2 & -x_3 & +x_4 & = 0 \\ 3x_1 & +x_2 & -2x_3 & +2x_4 & = 3 \end{array}$$

   Check the ranks of the coefficent matrix and augmented matrix. How would you interpret the results? Try to solve the system in MATLAB using \ operator and Moore-Penrose pseudo inverse *pinv*. Compute the norms of residual vectors $\mathbf{r} = \mathbf{A}\mathbf{x} - \mathbf{b}$. Compare the results obtained with different methods. How would you interpret the result?

6. Generate random binary matrix $\mathbf{A}$ of size $m \times n$ and a random vector $b \in \mathbf{R}^m$. Store $\mathbf{A}$ and $\mathbf{b}$ to the disk in ASCII format (*save*). Using some text editor, write a MATLAB function that takes in one argument v. First the function loads $\mathbf{A}$ and $\mathbf{b}$ from the disk (*load*). Then, if $v = 0$, the function solves a linear equation system $\mathbf{A}\mathbf{x} = \mathbf{b}$ by applying \ operator and returns $\mathbf{x}$. If $v <> 0$ the function returns the augmented matrix $(\mathbf{A}|\mathbf{b})$ in reduced row echelon form. (Hint: type *help function* to MATLAB prompt.)